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"Investigation of Surface-Scattering Losses of III-V Compound Semiconductors"

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10 June 1981

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GaAs, GaP, Surface Scattering, electrooptics

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Stages 1 and 2 of the proposed program were completed during the first 18 months of this effort. A satellite chamber for device fabrication was designed, built and incorporated into the main high vacuum chamber. The free surface properties of GaAs (110) and GaP (110) were studied using LEED and AES. Reports on this work were presented at scientific meetings. The problem of a wave propagating with losses in a rectangular waveguide was formulated and solved.

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A

RESEARCH ACCOMPLISHMENT IN THE FIRST YEAR

The research program was planned in four stages over two calendar years starting September 28, 1979. The first and second stages would encompass both the investigation of free surface properties of the semiconductors in an altra-high vacuum (10^{-10} torr) system (UHV) equipped with LEED, Auger electron spectroscopy (AES) and ion-sputtering capabilties, and the design and assembly of the satellite chamber for device fabrication. These two stages would be performed at the same time without interference with each other. The third stage of the work would encompass the study of the waveguide surface structure, formulation of surface-scattering theory and characterization of scattering loss parameters in terms of the surface geometry through LEED and SEM analysis in the same ultrahigh vacuum system. The last stage of the work would encompass the fabrication of the oxide-free electro-optical devices, the measuring of the optical propagation losses and the study of the optical characteristics of such waveguides.

The first and the second stages of the planned research were completed. Currently we are conducting the third stage of the research plan. The satellite chamber for device fabrication, as shown in Fig. 1, was designed and assembled to the main system. A schematic diagram of the main system is shown in Fig. 2. The satellite chamber is indicated as subsystem 2 in Fig. 2. Subsystem 1 in Fig. 2 is a plasma reactor designed for oxidation research of GaAs and is in no interference with this research program. A photograph of the completed system is shown in Fig. 3.

The free surface properties of GaAs (110), GaP (110) have been studied by LEED and AES. Results of the studies have been presented in two different scientific conferences. Abstracts, which summarized the results of the research works, are given in Appendices A and B. Abstract of the first paper entitled "Chain Method of LEED/MEED Intensity Calculation for Diatomic Surface" is given in Appendix A

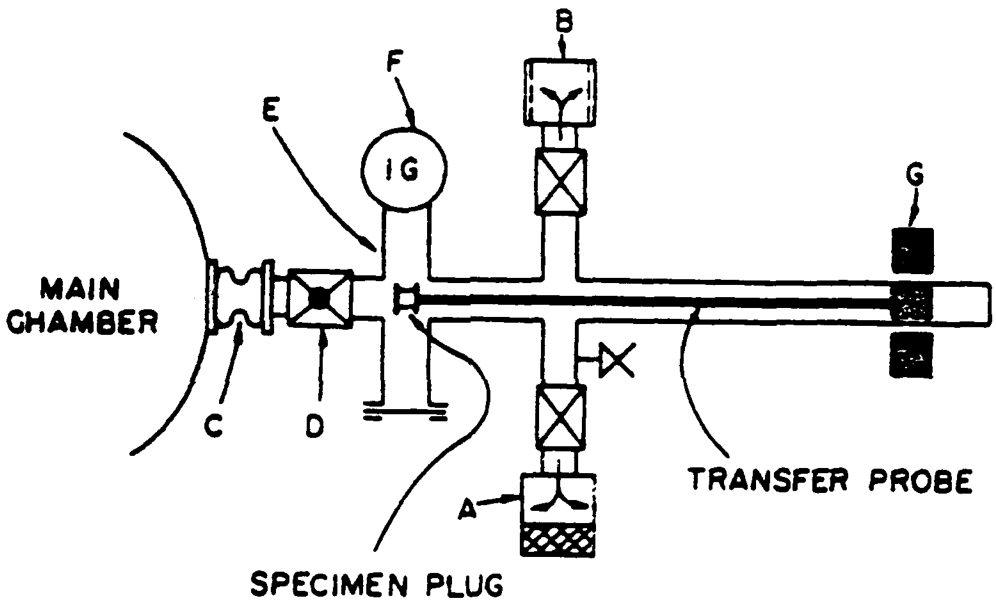

Fig. 1. Top-view schematic of the satellite system attached to the main chamber (left). A: sorption forepump; B: Vacion UHV pump; C: bellows-coupled flange; D: isolation valve; E: six-armed cross; F: ionization gauge; G: transfer probe magnet actuator. In the schematic the transfer probe is shown completely retracted, with the specimen plug located at the center of the six-armed cross.

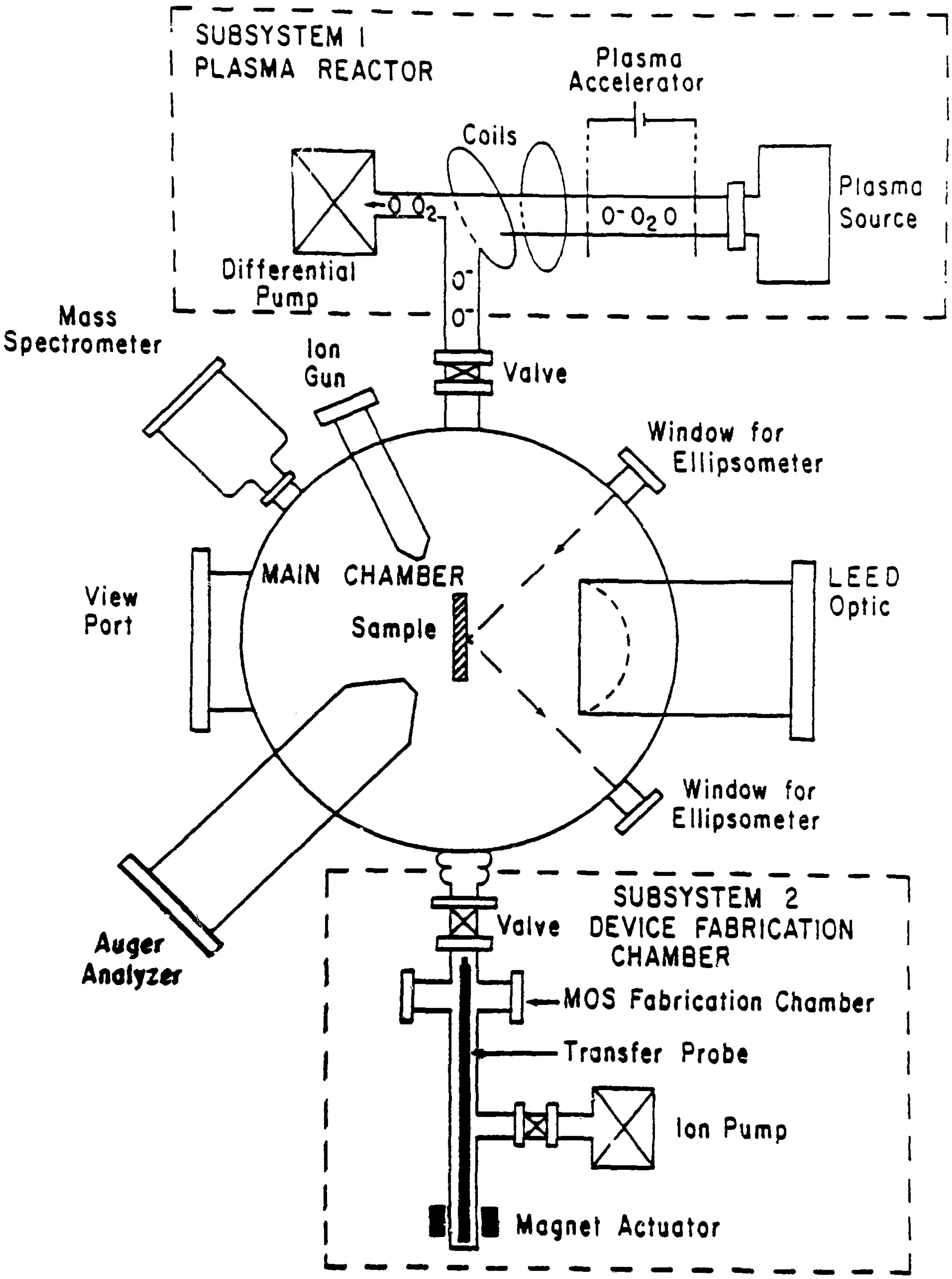

Fig. 2. Schematic diagram of the UHV system equipped with LEED, AES, mass spectrometer, ion sputtering gun and ellipsometer for surface analysis. Subsystem 1 is the plasma reactor which is currently under construction. Subsystem 2 is the MOS device fabrication chamber.

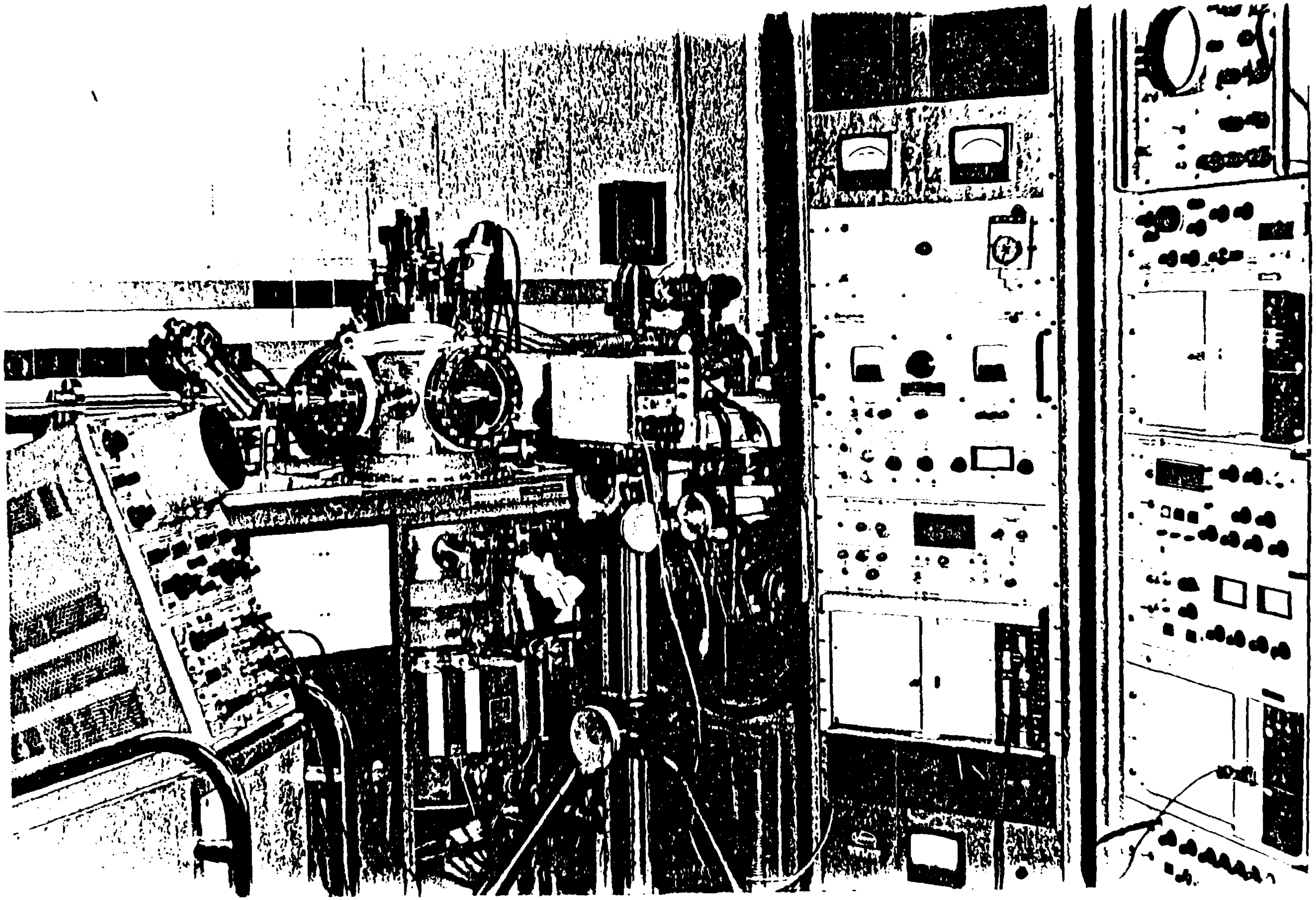

Fig. 3. LEED/AES system for investigation of surface and interface properties of semiconductors and ceramic materials. The capital equipment is acquired through the NSF equipment grant.

and the second paper entitled "Atomic Structure of GaAs (110) Face" is given in Appendix B. Both papers are being prepared for publication.

The third stage of the research work is currently ongoing. The theoretical formulation of surface scattering theory and characterization of the scattering loss parameters are currently being developed. The problem of a wave propagating along z direction in a rectangular dielectric waveguide with dimensions a and b as shown in Fig. 4 was formulated and solved. The wave functions in the waveguide were solved using predominantly polarized approximation.* The propagation attenuation constants, e.g. the loss parameters, were calculated. Details of the theoretical formulation and calculation is given in Appendix C.

Citations of research results, which are presented in conferences or submitted for publication, are listed in Appendix D.

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*D. Marcuse, Bell Syst. Tech. J., 48, 3187 (1969).

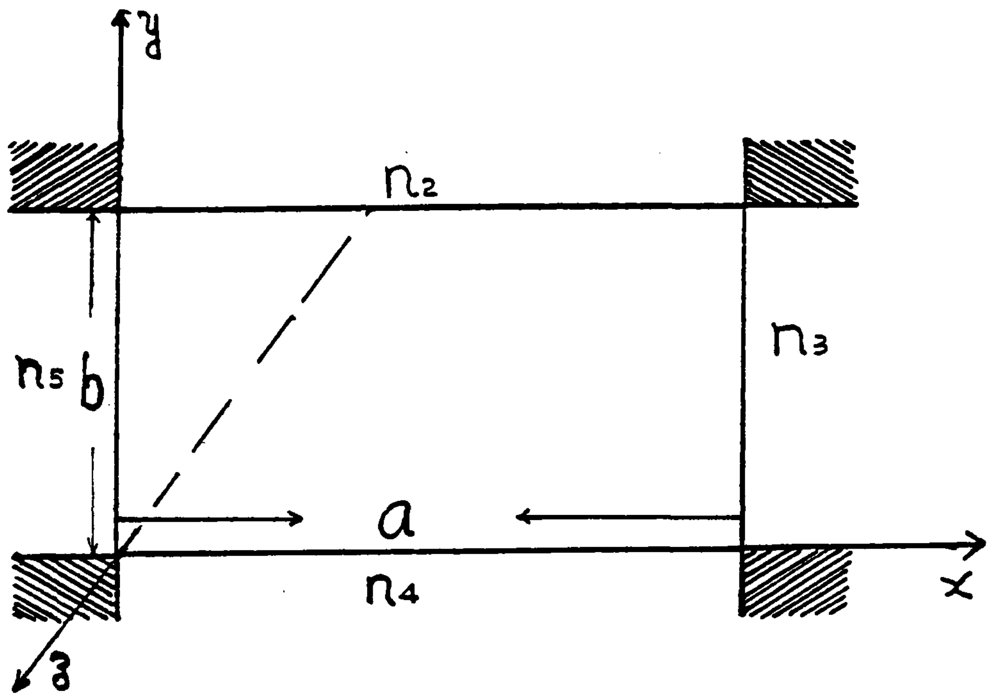

Fig. 4. Cross section of a rectangular dielectric waveguide with dimensions a and b. n_1, n_2, n_3, n_4 and n_5 are the indixes of refraction of regions 1 to 5, respectively.

APPENDIX A

The following paper was presented in the Conference on Determination of Surface Structure by LEED, IBM Thomas J. Watson Research Center, Yorktown Heights, N.Y., June 19-20, 1980

CHAIN METHOD OF LEED/MEED INTENSITY CALCULATION FOR DIATOMIC SURFACES

In low energy electron diffraction (LEED), crystal surface is probed by bombarding electrons of energy $E \leq 200$ eV at normal incidence to the surface and analyzing the intensity of those elastically scattered electrons which are back reflected from the surface, whereas in medium energy electron diffraction (MEED) due to an increase in the energy of incident electrons ($200 \text{ eV} < E \leq 5$ keV) the surface sensitivity of the technique is maintained by taking an oblique angle of incidence. Interpretation of LEED/MEED intensity spectra requires detailed calculations of the diffracted intensities for a series of trial models. For such a technique to work, a rapid and accurate method of calculation is needed.

Layer-KKR method is conventionally employed in the calculation of LEED intensity spectra. In this formalism, the crystal is divided into a number of layers parallel to the crystal surface and scattering calculations are split into two parts, intralayer and interlayer. In the intralayer scattering calculation, due to the assumed spherical symmetry of the atomic potential, an angular momentum representation is used. This involves large matrices and their inverses and becomes cumbersome with increasing energy and more complex surfaces. In such a situation we propose to use the chain method of intralayer multiple scattering calculation.

In the chain method the two dimensional intralayer multiple scattering calculations are further divided into two one dimensional steps, scattering within a chain of atoms and between the chains. At each stage of calculation the electron wave function is represented in terms of an appropriate set of basis functions. The scattering by an atom, a chain of atoms and a layer of chains is expressed in terms of spherical, cylindrical and plane wave representation respectively. As we move from one stage to another, transformation from one basis set to another is carried out. Due to its one dimensional lattice summations the chain scattering formalism has many computational advantages over the layer-KKR method both for normal incidence LEED and off normal incidence MEED.

APPENDIX B

The following paper is submitted for presentation in the Eighth Annual Conference on the Physics of Compound Semiconductor Interfaces, Williamsburg, Virginia, January 27-29, 1981.

ATOMIC STRUCTURE OF GaP (110) FACE

ABSTRACT

Low energy electron diffraction (LEED) intensities have been measured for the (110) face of GaP and analyzed using a dynamical multiple-scattering model of the diffraction process. The intralayer multiple scattering is treated exactly, while for the interlayer multiple scattering, the renormalized-forward-scattering method is used. Comparison of the calculated and observed LEED intensities suggests that both the Ga and the P atoms on the (110) face may exhibit a contracted outermost layer spacing. The surface layer is compressed by about 5% such that the top layer spacing is reduced by 0.1 ± 0.02Å. The rippled geometry of surface reconstruction is not clearly observed. This indicates that the GaP (110) surface atomic structure is different to that of GaAs (110). By comparing the structures and properties of the GaP (110) face and the GaAs (110) face, it is concluded that the GaP (110) face is relatively unstable and reactive.

APPENDIX C

The problem of a wave propagating along z-direction is a rectangular dielectric waveguide with dimension a and b as shown in Figure 4 is formulated and solved. An exact analytical treatment of rectangular waveguide is practically impossible. Therefore the approximated analytical approach developed by Marcatili (a) is followed in solving the problem.

Assuming a traveling wave propagating in z-direction as shown in Fig. 4, Maxwell equations

$$\nabla x\bar{H} = \varepsilon_o n^2 \frac{\partial \bar{E}}{\partial t} \qquad \nabla x\bar{E} = \mu_o \frac{\partial \bar{H}}{\partial t}$$

can be solved to give

$$E_x = [-i/(n^2k^2-\beta^2)][\beta \frac{\partial E_z}{\partial x} + \mu_o\omega \frac{\partial H_z}{\partial y}] \quad \text{- - - - - - - - - - - - - - - (1)}$$

$$H_g = [-i/(n^2k^2-\beta^2)][\beta \frac{\partial H_z}{\partial y} + \omega\varepsilon_o n^2 \frac{\partial E_z}{\partial y}] \quad \text{- - - - - - - - - - - - - - - (2)}$$

$$E_y = [-i/(n^2k^2-\beta^2)][\beta \frac{\partial E_z}{\partial y} - \omega\mu_o \frac{\partial H_z}{\partial x}] \quad \text{- - - - - - - - - - - - - - - (3)}$$

$$H_x = [-i/(n^2k^2-\beta^2)][\beta \frac{\partial H_z}{\partial x} - \omega t_o n^2 \frac{\partial E_z}{\partial y}] \quad \text{- - - - - - - - - - - - - - - (4)}$$

$$E_z = \frac{1}{\varepsilon_o n^2 i\omega} (\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) \quad \text{- - - - - - - - - - - - - - - (5)}$$

$$H_z = \frac{i}{\mu_o\omega} (\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}) \quad \text{- - - - - - - - - - - - - - - (6)}$$

where n is the index of refraction of the dielectric; k is the wavevector in free space; ω is the angular frequency of the wave and β is the propagation constant in z-direction.

Substituting (2) and (4) into (5), and (1) and (3) into (6), we have

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (n^2k^2-\beta^2)\, E_z = 0 \quad \text{- - - - - - - - - - - - - - - (7)}$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y} + (n^2k^2-\beta^2)\ H_z = 0 \qquad \text{- - - - - - - - - - - - - - - -} \quad (8)$$

Two different types of modes can be supported in the waveguide[b], e.g.,

E^x_{pq} : polarized predominately in x-direction and

E^y_{pq} : polarized predominately in y-direction,

where p and q are positive integers indicating the modes of the wave propagating in the waveguide.

E^x_{pq} MODE

In region 1 with refraction index n_1 as shown in Fig. 4, the waveequation (7) can be solved to give

$$E_z(x,y) = A \text{ Cos } k_x\ (x+\xi) \text{ Cos } k_y\ (y+\eta) \qquad \text{- - - - - - - - - - - - - - - -} \quad (9a)$$

with $H_x = 0$, where ξ and η are the phase factors of E_z.

Other components of the wavefunction can be obtained by substituting (9a) into eqs. (1), (3) and (4):

$$H_z = -A\frac{n_1^2}{\beta}\ \left(\frac{\varepsilon_o}{\mu_o}\right)^{1/2} \left(\frac{k_y}{k_x}\right) \text{ Sin } k_x\ (x+\xi) \text{ Sin } k_y\ (c+\eta) \text{ - - - - - - - - - -} \quad (9b)$$

$$E_x = iA\ \frac{(n_1^2k^2-k_x^2\ )}{\beta\ k_x} \text{ Sin } k_x\ (x+\xi) \text{ Cos } k_y\ (y+\eta) \qquad \text{- - - - - - - - - - - - - -} \quad (9c)$$

$$E_y = \frac{-iAk_y}{\beta} \text{ Cos } k_x\ (x+\xi) \text{ Sin } k_y\ (y+\eta) \qquad \text{- - - - - - - - - - - - - - - -} \quad (9d)$$

Substituting (9a) into (7) we have

$$n_1^2k^2-\beta^2 = k_x^2 + k_y^2 \qquad \text{- -} \quad (10)$$

For small incident angle, $\beta >> (k_x^2 + k_y^2)$, eq. (10) can be approximated to be $\beta \doteq n_1 k \cos\theta \doteq n_1 k$ for $\theta < 5°$.

The wavefunctions in regions 2, 3, 4 and 5 with indices of refraction n_2, n_3, n_4 and n_5, respectively, as shown in Fig. 4, are solved. When boundary conditions are matched between region 1 and the neighboring regions, i.e., regions 2, 3, 4 and 5, we obtain the four transdental equations:

$$\tan k_x a = n_1^2 k_x \, (n_3^2 \gamma_5 + n_5^2 \gamma_3)/(n_3^2 n_5^2 \, k_x^2 - n_1^4 \gamma_3 \gamma_5) \qquad (11a)$$

$$\tan k_x \xi = \left(\frac{n_5}{n_1}\right)^2 (k_x/\gamma_5) \qquad (11b)$$

$$\tan k_y b = k_y \, (\gamma_2 + \gamma_4)/(k_y^2 - \gamma_2 \gamma_4) \qquad (12a)$$

$$\tan k_y \eta = -\gamma_4/k_y \qquad (12b)$$

where $\gamma_2 = [(n_1^2 - n_2^2)k^2 - k_y^2]^{1/2}$

$$\gamma_3 = [(n_1^2 - n_3^2)k^2 - k_x^2]^{1/2}$$

$$\gamma_4 = [(n_1^2 - n_4^2)k^2 - k_y^2]^{1/2}$$

$$\gamma_5 = [(n_1^2 - n_5^2)k^2 - k_x^2]^{1/2}$$

A computer program is currently being developed using Newton's method to solve equations (11) and (12) to obtain k_x, k_y, β, and the power attenuation constant α.

REFERENCES:

(a) Marcatili, E. A. J., Bell Syst., Tech. J. 48, 2071-2102 (1969).

(b) D. Marcuse, "Theory of Dielectric Optical Waveguide", Academic Press, New York, 1974.

(c) I. P. Kaminow, W. L. Mammel, and H. P. Weber, Applied Optics, Vol 13, No. 2/ Feb. 1974.

APPENDIX D

Publication citations and presentations supported or partially supported by this research grant:

1. "Decomposition of Aluminum oxide by Electron Bombardment." B. W. Lee and J. M. Kuo, B. Am. Phys. S., 25 (3), 238, 1980.

2. "Study of MIS Polycrystalline Silicon Solar Cell Using Auger Electron Spectroscopy," J. M. Kuo, B. W. Lee, B. Lalevic and W. A. Anderson, B. Am. Phys. S., 25 (3), 409, 1980.

3. "Chain Method of LEED/MEED Intensity Calculation for Diatomic Surfaces," N. Masud, C. G. Kinniburgh, D. J. Titterington. Presented in Conference on Determination of Surface Structure by LEED, IBM, T. J. Watson Research Center, Yorktown Hights, NY, June, 1980.

4. "Study of MIS Silicon Cell by ESCA and AES," Y. S. Wang, H. J. Yu, C. C. Hsu, B. W. Lee and W. A. Anderson, presented in the 27th National Symposium of American Vacuum Society, Detroit, October, 1980. Submitted to J. Vac. Sci. Technol for publication.

5. "Stability of MIS Silicon Solar Cell," B. W. Lee, J. M. Kuo, B. Lalevic and W. A. Anderson, submitted to J. Vac. Sci. Technol. for publication.

6. "Atomic Structure of GaP(110) Face," B. W. Lee, R. K. Ni and N. Masud, submitted for presentation in the Eighth Annual Conference on the Physics of Compound Semiconductor Interfaces, Williamsburg, Virginia, Jan. 1981.

A NONLINEAR MAXIMUM ENTROPY METHOD FOR SPECTRAL ESTIMATION

Summary

An intensive research study is made of the nonlinear maximum entropy spectral analysis method proposed by P. F. Fougere of AFGL. The research not only provides a much better understanding of the properties of the method including spectral resolution, convergence, etc. but also shows that the method can be performed effectively with the PDP 11/45 minicomputer. The successful implementation of the method is described in the program listings in Appendix I. Extensive computer results are presented on various data. These results clearly confirm that the nonlinear method is superior to the Burg's maximum entropy spectral analysis.

Preliminary results on the multichannel (multivariate) maximum entropy spectral analysis, the two-dimensional maximum entropy spectral analysis, and computer graphics for the spectral display are also presented. An extensive bibliography of the maximum entropy spectral analysis is given in Appendix III.

I. Review of Research Progress

This research is concerned with the nonlinear maximum entropy spectral analysis (MESA) method proposed by Dr. Paul Fougere of AFGL. As verified in this research work performed at the PDP 11/45 minicomputer, the method not only provides a much better spectral resolution than the Burg's method but also removes the line-splitting and frequency shifting phenomena for sinusoidal signals, as experienced in the Burg's method. By using the double precision, the minicomputer results are reasonably close to those obtained at the CDC 6600 computer at AFGL.

The final computer program developed for the nonlinear complex signal maximum entropy spectral analysis is shown in Appendix IA. The program follows the mathematical development of Fougere [1] but is quite different from the original computer program developed by Dr. Fougere. Appendix IB is the computer program for Burg's complex signal maximum entropy spectral analysis. By using the computer programs and the two-channel radar data as shown in Fig. 1 (see also [2]), the spectrum of the nonlinear method is shown in Fig. 2a (linear plot) and Fig. 2b (logarithmic plot) for 10 filter weights. The Burg's result is shown in Fig. 3a (linear plot) and Fig. 3b (logarithmic plot), also for 10 filter weights. More detailed tabulation of the major frequency components outside the clutter bandwidth (-0.167fs, +0.167fs) is given in Fig. 4. The nonlinear method which is very close to the true answer clearly is much better than the Burg's method. 30 iterations are used in the nonlinear method which appears to be an optimum

number. The optimum filter weight is around 10 or 11 as determined by Fig. 5 which shows the finear prediction error (FPE) according to the Akaike criterion. The solid curve is the lower bound and the dashed curve is the upper bound as tabulated in Fig. 6. The upper bound was originally proposed in the research proposal[2] and it seems to be better than the lower bound due to Akaike.

Major documentations already made which describe the research progress are as follows:

1. C. H. Chen, J. Chen, and C. Yen, "A minicomputer implementation of Fougere's maximum entropy spectral analysis method," Technical report prepared for the Mini-grant, August 20 1980. This report has detailed results of comparison between Burg's and the nonlinear methods for sinewave and sunspot data.
2. C. H. Chen, "Spectral resolution of Fougere's maximum entropy spectral analysis," to be published in the Proceedings of IEEE, June 1981. This journal paper based on the work performed under the Mini-grant provides a good comparison between the nonlinear and Burg's methods, and the other method for complex sinusoids. The Cramer-Rao bound is used as a reference.
3. C. H. Chen and C. Yen, "Note on computer graphics for maximum entropy spectral analysis," Technical report prepared for the Mini-grant, March 23, 1981. This report provides a three-dimensional spectral display of sinewaves for both nonlinear and Burg's methods.

A number of important results are included in the new research proposal submitted to AFOSR in December 1980. Appendix III provides an extensive list of references on the maximum entropy spectral analysis. The following sections describe some new research areas with preliminary results.

II. Multichannel (multivariate) Maximum Entropy Spectral Analysis

A mathematical presentation of this topic is given in Appendix II. Several computer programs for multichannel maximum entropy spectral analysis were provided by Dr. Fougere. The following results are based on the time series of sunspot numbers, northern light activity, and earthquake activity by using the third multichannel program. The data are tabulated in [3].

Fig. 7a is the first channel (sunspot number) auto-spectrum with linear (left) and logarithmic (right) scales, and 16 lags.

Fig. 7b is the second channel (northern light activity) auto-spectrum with linear(left) and logarithmic (right) scales, and 16 lags.

Fig. 7c is the third channel (earthquake activity) auto-spectrum with linear (left) and logarithmic (right) scales and 16 lags.

Fig. 7d is the cross-spectrum between channels 1 and 2 with real part (left) and imaginary part (right), and 16 lags.

Fig. 7e is the cross-spectrum between channels 2 and 3 with real part (left) and imaginary part (right), and 16 lags.

Fig. 7f is the cross-spectrum between channels 1 and 3 with real part (left) and imaginary part (right), and 16 lags.

By way of verification, it is interesting to note that the spectral peak for the sunspot numbers is determined accurately.

III. Two-Dimensional Maximum Entropy Spectral Analysis

We consider a very simple separable case in the two-dimensional spectral analysis. The signal considered is $\sin(2\pi x)\sin(2\pi y)$. In this case the power spectrum is the product of the power spectra

of $\sin(2\pi x)$ and $\sin(2\pi y)$. In each spatial dimension, the power spectrum can be determined by using the Fourier, nonlinear and Burg's methods. The two-dimensional spectra are shown in Figs. 8, 9 and 10 respectively, based on the Fourier, nonlinear, and Burg's methods. The nonlinear method clearly is much better. Extension of the above procedure to a more general two-dimensional spectral analysis is not possible. Although some two-dimensional maximum entropy spectral analysis work has been reported (see Appendix III), the success is very limited. Further research is much needed.

IV. Conclusions and Recommendations

The nonlinear maximum entropy spectral estimation method proposed by P. F. Fougere has provided superior spectral estimation over the Burg's method in a number of data considered. The computational requirement of the nonlinear method is, however, significantly higher. Typical number of iterations needed is 20 to 30. The use of minicomputer has not created significant computational problem as predicted. It is our strong belief that the nonlinear method will become very popular in high resolution spectral analysis for a wide range of applications in geophysics, sonar, radar areas, etc.

Recommended further studies include the multichannel (multivariate) maximum entropy spectral analysis, the two-dimensional nonlinear maximum entropy spectral analysis, computer graphics for the spectral display, signal decomposition, and

signal prediction and extrapolation.

Acknowledgment

We would like to thank Dr. Henry R. Radoski for support and encouragement, and Dr. Paul F. Fougere for guidance and many helpful discussions.

References

1. P. F. Fougere, "A solution to the problem of spontaneous line splitting in maximum entropy power spectrum snalysis of complex signals," Proc. of the RADC Spectrum Estimation Workshop, May 1978.

2. C. H. Chen, Mini-grant Proposal on "A Non-Linear Maximum Entropy Method for Spectral Estimation" for AFOSR, Sept. 1979.

3. E. A. Robinson, "Multichannel Time Series Analysis with Digital Computer Programs," Holden-Day, San Francisco, 1967.

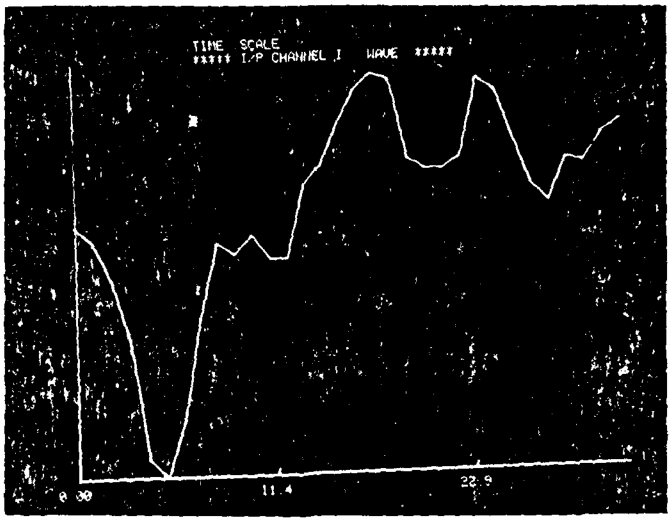

Fig. 1a 32-point I-channel
data, for real part.

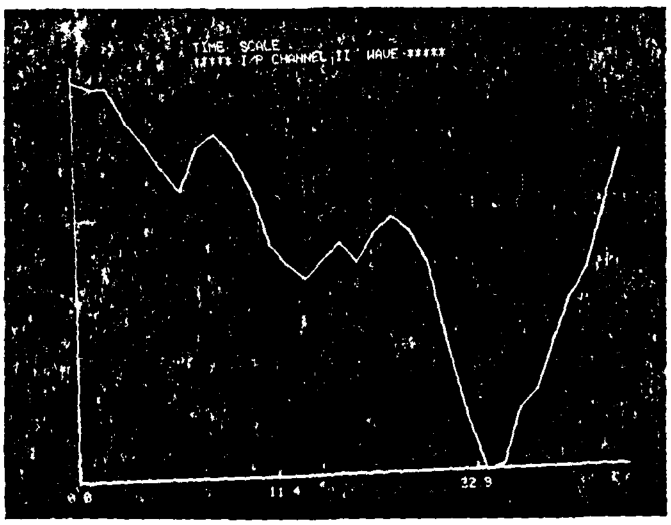

Fig. 1b 32-point Q-channel
data for imaginary
part.

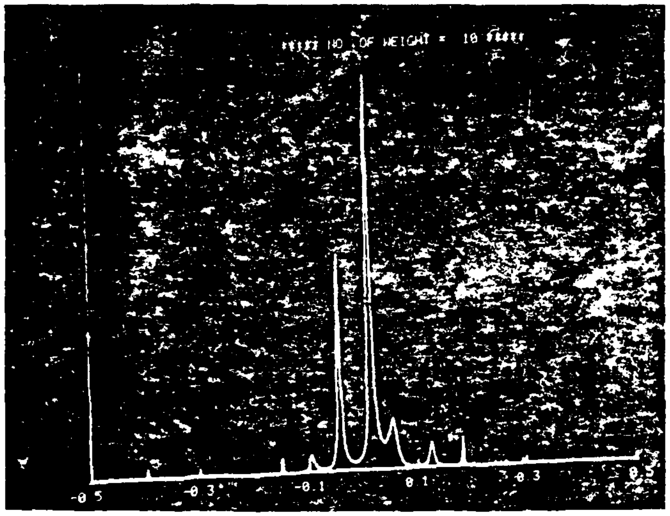

Fig. 2a Nonlinear MESA (linear plot)

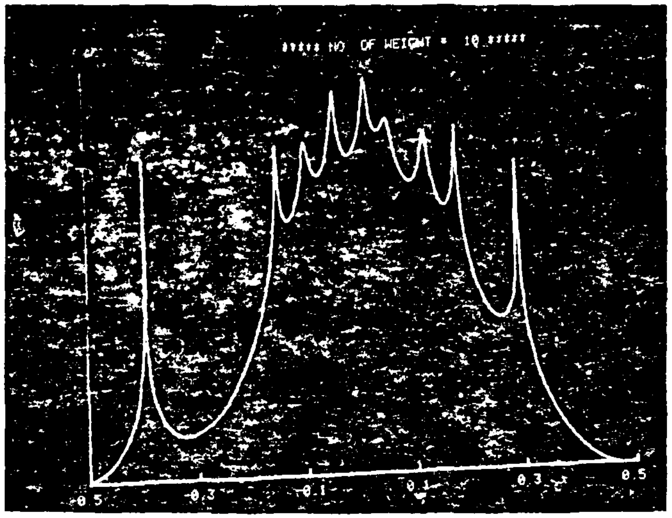

Fig. 2b Nonlinear MESA (log plot)

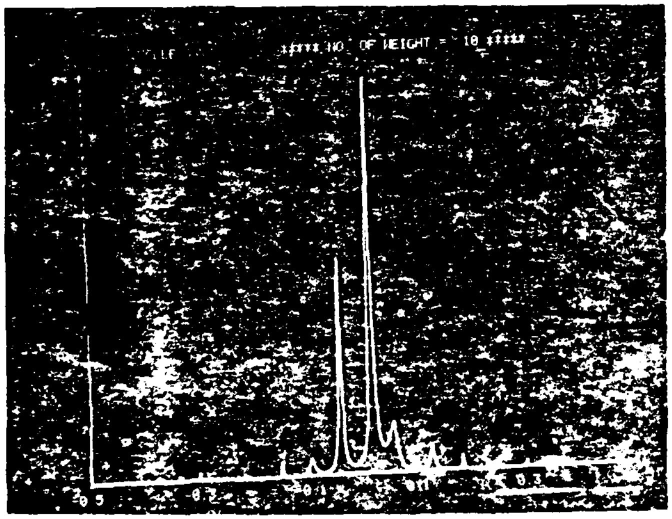

Fig. 3a Burg's MESA (linear plot)

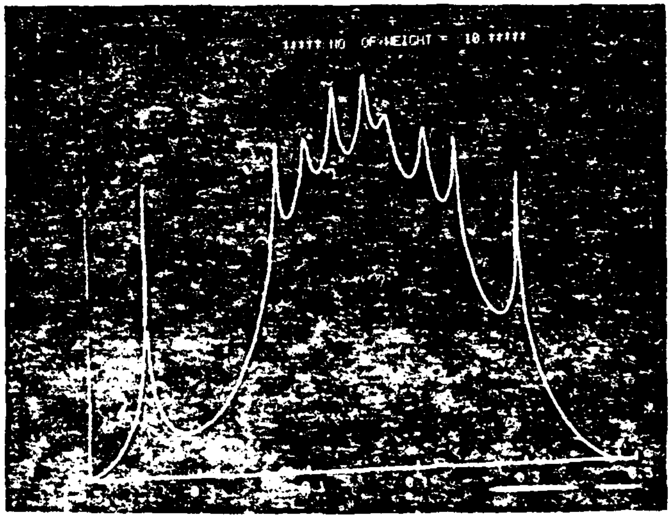

Fig. 3b Burg's MESA (log plot)

Frequency	Amplitude	Relative level	
+0.1836 f_s	0.2184	0	dB
-0.3945 f_s	0.1203	-5.18	dB
+0.2930 f_s	0.0873	-7.96	dB

True Answer

Frequency	Amplitude	Relative level	
+0.18359 f_s	5.48007	0	dB
-0.39453 f_s	3.02523	-5.16	dB
+0.29296 f_s	2.19424	-7.95	dB

Result of Nonlinear Method

Frequency	Amplitude	Relative level	
+0.18164 f_s	3.58956	0	dB
-0.39453 f_s	1.45378	-7.85	dB
+0.29492 f_s	1.40613	-8.14	dB

Result of Burg's Method

Figure 4

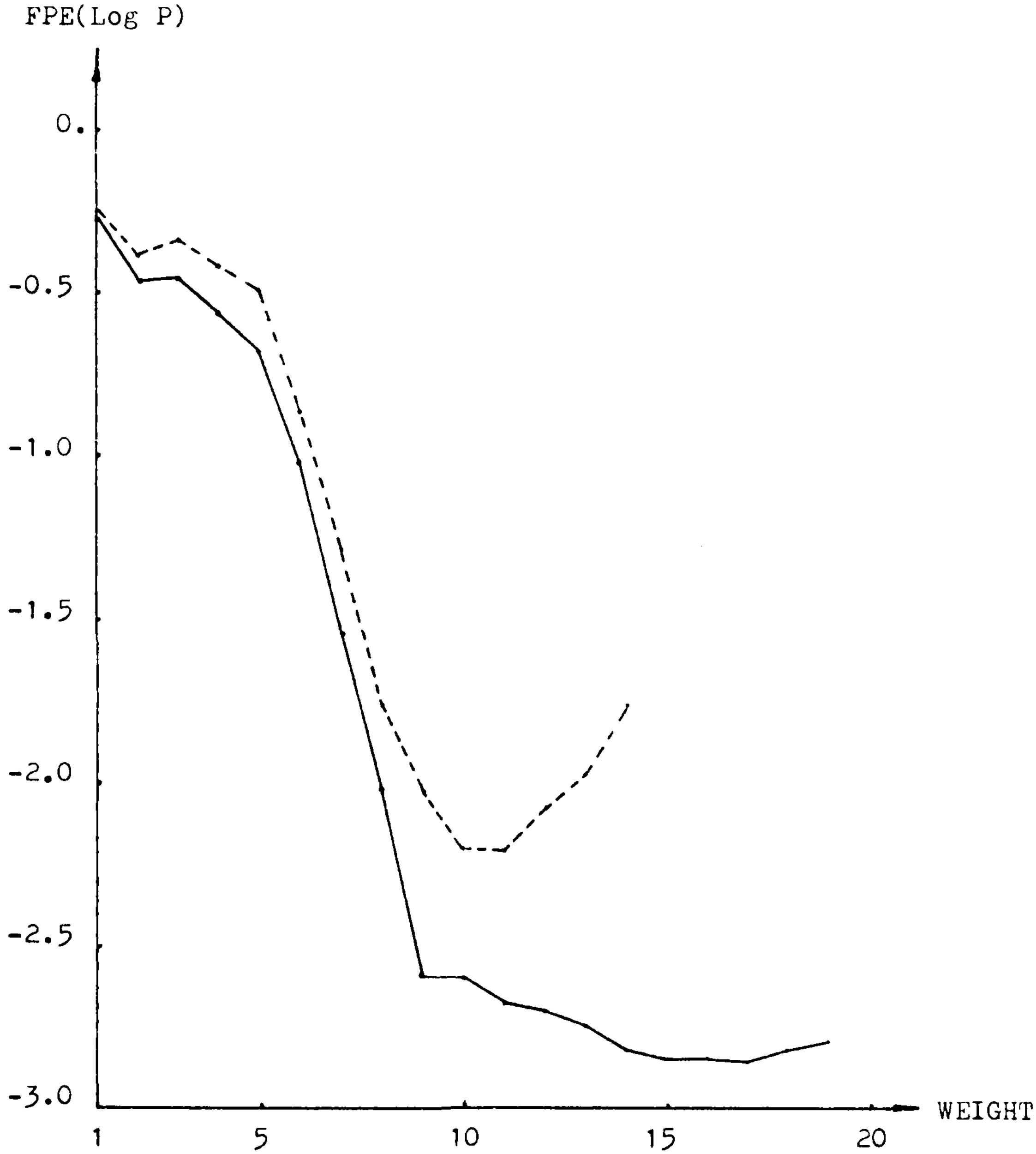

Figure 5

No. of weight	Log. of $\frac{N+(M+1)}{N-(M+1)} \cdot P_m$	Log. of $\frac{N+2(M+1)}{N-2(M+1)} \cdot P_m$
1	-0.279	-0.224
2	-0.463	-0.381
3	-0.445	-0.332
4	-0.562	-0.418
5	-0.674	-0.496
6	-1.080	-0.866
7	-1.539	-1.283
8	-2.063	-1.762
9	-2.587	-2.006
10	-2.597	-2.175
11	-2.678	-2.176
12	-2.686	-2.075
13	-2.738	-1.971
14	-2.815	-1.764
15	-2.839	-
16	-2.824	-
17	-2.833	-
18	-2.810	-
19	-2.799	-
	Lower Bound	Upper Bound

Note: N is the number of data points in each channel (32 in this case)

M is the number of filter weights

P_m is the error power

Figure 6

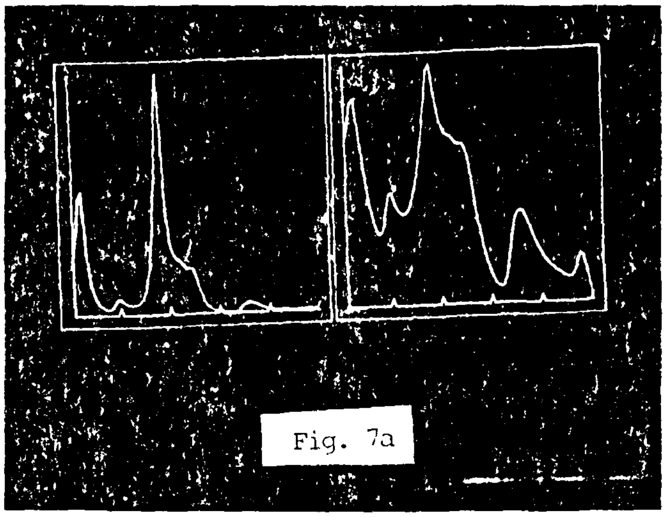

Fig. 7a

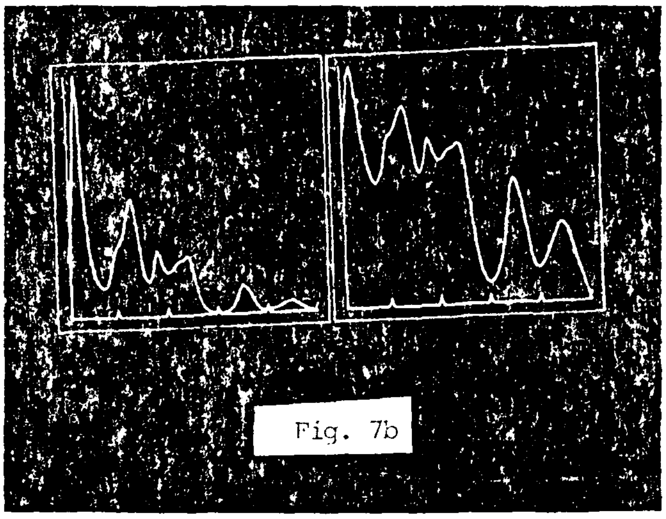

Fig. 7b

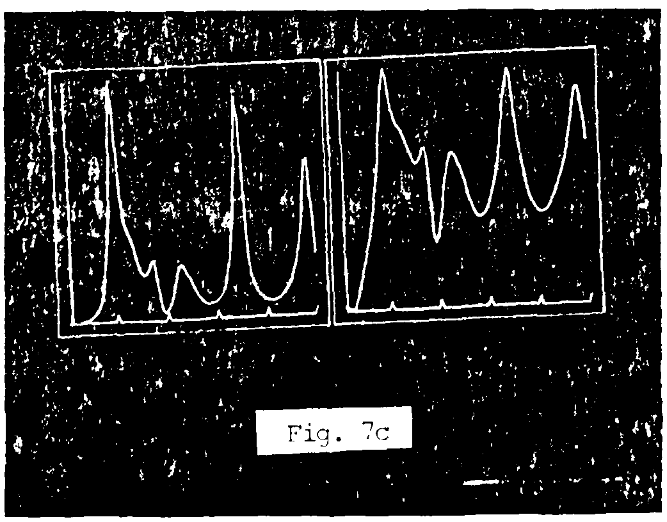

Fig. 7c

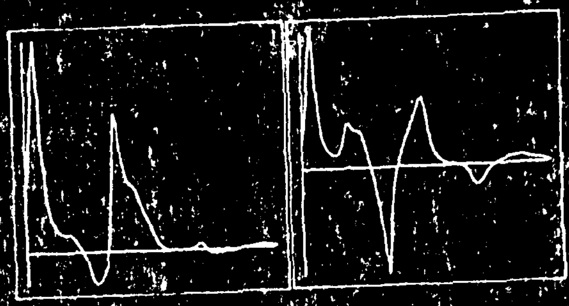

Fig. 7d

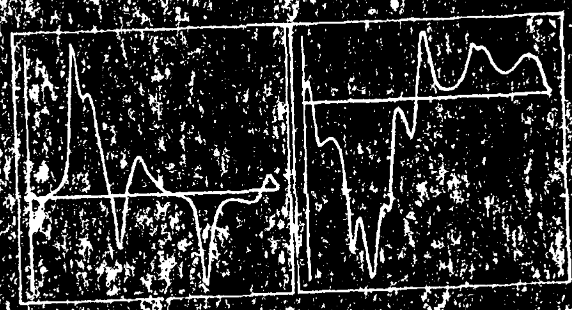

Fig. 7e

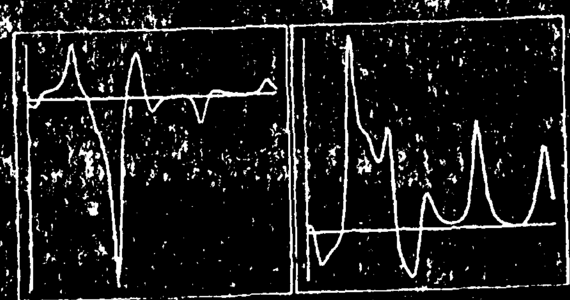

Fig. 7f

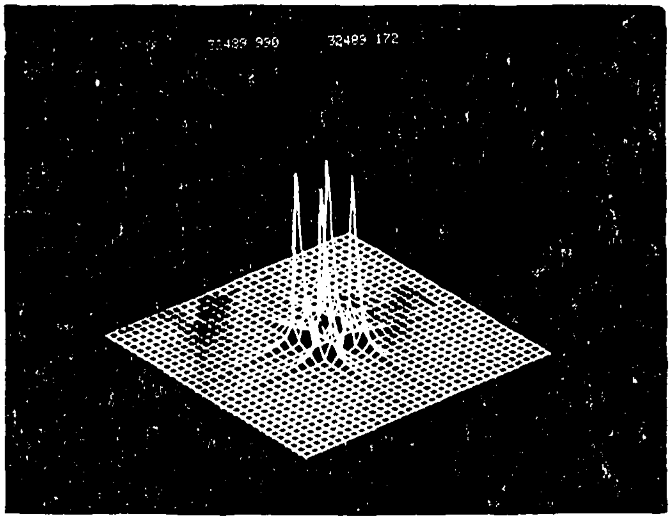

Fig. 8a (linear plot)

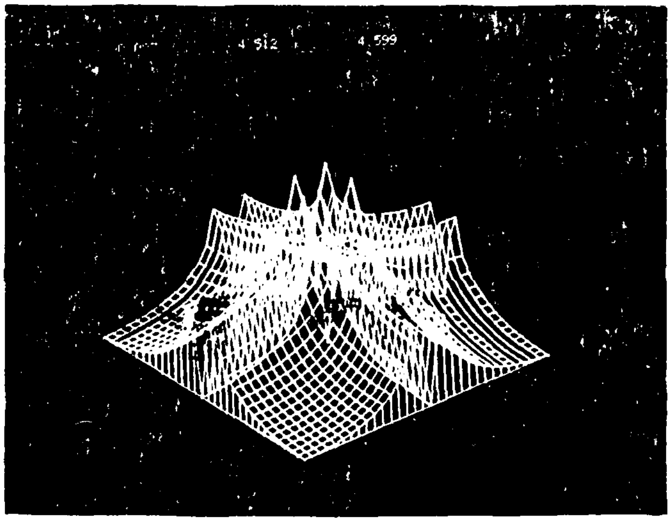

Fig. 8b (logarithmic plot)

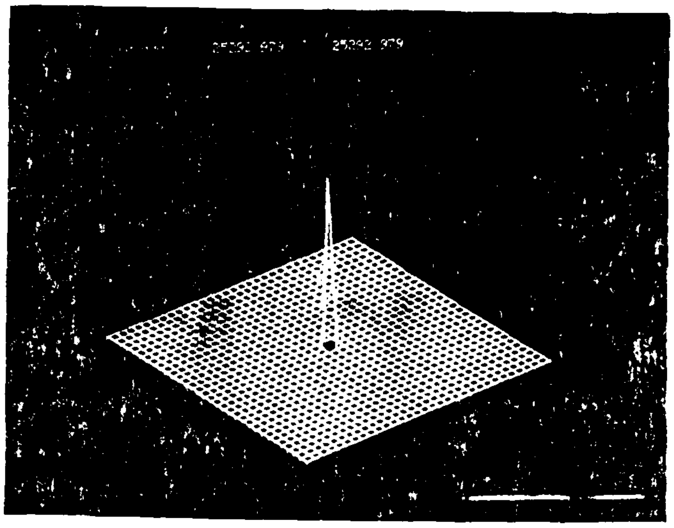

Fig. 9a (linear plot)

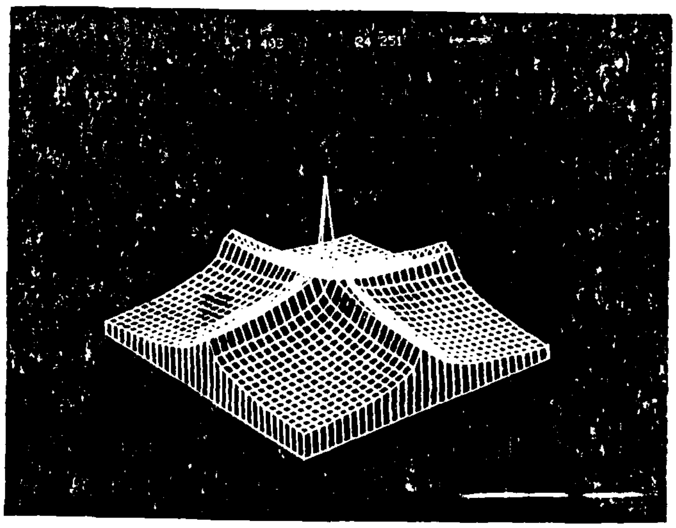

Fig. 9b (logarithmic plot)

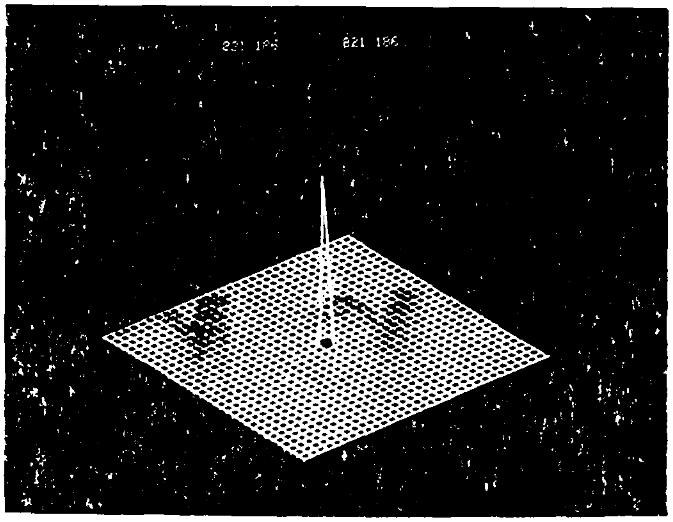

Fig. 10a (linear plot)

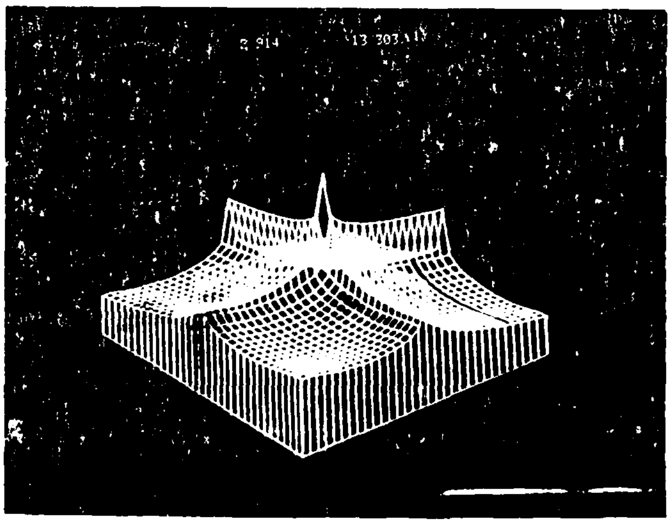

Fig. 10b (logarithmic plot)

Appendix IA Nonlinear Complex Signal MESA Program Listing

```
C
C       NAME: OPTMR [illegible]       MAX. ENTROPY [illegible]
C
        DOUBLE PRECISION XR(32), XI(32), [illegible]
        REAL XB([illegible])
        COMMON [illegible]
        DATA NF/[illegible]/
        READ(6,10)LX,LA,ICA,IMI,LG,[illegible]
10      FORMAT(5I5,F12.5,D13.8)
        DEFINE FILE 2([illegible],U,LINE)
        LINE=1
        READ(2'LINE)(XB(K),K=1,LX)
        DO 15 I=1,LX
15      XR(I)=XB(I)
        LINE=2
        READ(2'LINE)(XB(K),K=1,LX)
        DO 25 I=1,LX
25      XI(I)=XB(I)
        END FILE 2
        DO 20 I=1,NF
20      XB(I)=FLOAT(I)
        DO 30 I=1,LX
30      CAL(I)=XR(I)
        CALL KBPLOT(XB,CAL,LX,10,1000,[illegible])
        DO 40 I=1,LX
40      CAL(I)=XI(I)
        CALL KBPLOT(XB,CAL,LX,10,1000,50,350)
        CALL BELL
        READ(6,50)[illegible]
50      FORMAT(10I10)
        CALL BURG(ICA)
        IF(LG.GE.0)CALL MESG
        IF(ICA.EQ.0)GO TO 60
        CALL OPTM(IMI,LG)
        CALL [illegible]
        IF(LG.GE.0)CALL MESG
60      CALL SPEC([illegible],NF)
        DO 70 I=1,NF
70      CAL(I)=EX*A(500-I)
        CALL KBPLOT(XB,CAL,NF,10,1000,[illegible])
        DO 80 I=1,NF
        CAL(I)=0
80      IF(MOD(I,25).EQ.1)CAL(I)=[illegible]
        CAL(1)=40
        CALL KBPLOT(XB,CAL,NF,10,1000,[illegible],750)
        CALL BELL
        CALL EXIT
        END
```

```
      SUBROUTINE BURG(ICA)
      DIMENSION H(10),PEF(32),PER(32),F(32),G(10),GGG(10)
      COMPLEX G,PEF,PER,H,GGG,F,P,D,SN,SD
      DOUBLE PRECISION XR(32),XI(32),AR(10),AI(10),GR(10,10
     1),GI(10,10),U,PM,POWER,THETA(20),TMPR,THE,EXTA([illegible]
      COMMON ZERO/NUM,LA,U,PM,POWER/BLK[illegible]
      EQUIVALENCE (EXTA(1),GR(1,1)),(EXTA(101),GI(1,1)),
     1(EXTA(201),AR(1)),(EXTA(211),AI(1)),(EXTA(221),THETA(1)),
     2,(EXTA(241),PEF(1)),(EXTA([illegible]),PER(1)),(EXTA([illegible]),F(1)),
     3(EXTA([illegible]),H(1)),(EXTA([illegible]),G(1)),[illegible]
      PM=POWER
      DO 10 I=1,NUM
10    F(I)=CMPLX(SNGL(XR(I)),SNGL(XI(I)))
      DO 110 NN=1,LA
      ZERO=(0.,0.)
      N=NN-1
      IF(N.EQ.0)GO TO 30
      DO 20 J=1,NUM
      PEF(J)=ZERO
20    PER(J)=ZERO
30    SN=ZERO
      SD=ZERO
      JJ=NUM-N-1
      DO 40 J=1,JJ
      D=F(J+N+1)+PEF(J)
      P=F(J)+PER(J)
      SN=SN-CONJG(P)*D
      SD=SD+CONJG(P)*P+D*CONJG(D)
40    CONTINUE
      GGG(NN)=2.*SN/SD
      GR(NN,NN)=REAL(GGG(NN))
      GI(NN,NN)=AIMAG(GGG(NN))
      TMPR=CABS(GGG(NN))
      IF(TMPR.GE.1.)TMPR=.99999
      IF(ICA.EQ.0)GO TO 50
      THE=TMPR/U
      THETA(NN)=DATAN(THE/DSQRT(1.D0-THE*THE))
      THETA(NN+LA)=DATAN2(GI(NN,NN),GR(NN,NN))
50    PM=PM*(1.-TMPR*TMPR)
      IF(N.EQ.0)GO TO 80
      DO 60 J=1,N
      K=N-J+1
      H(J)=GGG(J)-GGG(NN)*CONJG(GGG(K))
60    CONTINUE
      DO 70 J=1,N
70    GGG(J)=H(J)
      JJ=JJ-1
80    DO 90 J=1,JJ
      PER(J)=PER(J)+CONJG(GGG(NN))*(PEF(J)+F(J+NN))
      PEF(J)=PEF(J+1)+GGG(NN)*(PER(J+1)+F(J+1))
90    CONTINUE
      DO 100 I=1,NN
100   G(I)=GGG(I)
110   CONTINUE
      DO 120 I=1,LA
      AR(I)=REAL(G(I))
120   AI(I)=AIMAG(G(I))
      RETURN
      END
```

```
      SUBROUTINE SPEC(U,NF)
      DOUBLE PRECISION AR(10),AI(10),PM,DT,ANG,U,EXTS,TPI,EXTA(820)
     1,F,FINC,SCSR,SCSI
      COMMON /BLK02/ X,M,U,PM,EXTS /BLK05/EXTA
      EQUIVALENCE (EXTA(201),AR(1)),(EXTA(211),AI(1))
      TPI=8.D0*DATAN(1.D0)
      F=-1./4./DT
      FINC=1./DT/((NF-1)*2.)
      DO 5 I=1,NF
      SCSR=1.D0
      SCSI=0.D0
      DO 10 K=1,M
      ANG=-TPI*K*F*DT
      SCSR=SCSR+AR(K)*DCOS(ANG)-AI(K)*DSIN(ANG)
10    SCSI=SCSI+AR(K)*DSIN(ANG)+AI(K)*DCOS(ANG)
      EXTA(600+I)=PM*DT/(SCSR*SCSR+SCSI*SCSI)
5     F=F+FINC
      RETURN
      END



      SUBROUTINE BURG(ISW)
      DOUBLE PRECISION XR(32),XI(32),FR(32),FI(32)
     + ,BR(32),BI(32),THETA(20),AR(10),AI(10),
     + GR(10,10),GI(10,10),PS(20),PGPR(10,10),
     + PGPI(10,10),PGMR(10,10),PGMI(10,10)
     + ,U,PM,TMPR,TMPI,TMQR,TMQI,EXTA(820)
     + T2R,T2I,PTR,PTI,P2R,P2I,POWER,EXTS(50)
      COMMON /BLK02/M,U,PM,POWER/BLK1/XR,XI/BLK5/EXTA
      EQUIVALENCE (EXTA(1),GR(1,1)),(EXTA(101),GI(1,1)),
     + (EXTA(201),AR(1)),(EXTA(211),AI(1)),(EXTA(221),THETA(1)),
     + (EXTA(241),PS(1)),(EXTA(261),FR(1)),(EXTA(293),FI(1)),
     + (EXTA(341),BR(1)),(EXTA(373),BI(1)),
     + (EXTA(421),PGPR(1,1)),(EXTA(521),PGPI(1,1)),
     + (EXTA(621),PGMR(1,1)),(EXTA(721),PGMI(1,1))
      DO 310 J=1,M
      GR(J,J)=U*DSIN(THETA(J))*DCOS(THETA(M+J))
310   GI(J,J)=U*DSIN(THETA(J))*DSIN(THETA(M+J))
      DO 320 J=2,M
      J1=J-1
      DO 320 K=1,J1
      CALL CM(GR(J,J),GI(J,J),GR(J-1,J-K),GI(J-1,J-K)
     + ,TMPR,TMPI)
320   CALL CAD(GR(J-1,K),GI(J-1,K),TMPR,TMPI,
     + GR(J,K),GI(J,K))
      IF(ISW.EQ.0)GO TO 327
      DO 325 J=1,M
      AR(J)=GR(M,J)
325   AI(J)=GI(M,J)
      RETURN
327   PM=0.D0
      NM=N-M
      DO 410 K=1,NM
      FR(K)=XR(K+M)
      FI(K)=XI(K+M)
      BR(K)=XR(K)
      BI(K)=XI(K)
```

```
      DO 420 K=1,M
      CALL CM(XR(K+M+1),XI(K+M+1),GR(M,1),GI(M,1)
     - ,TMPR,TMPI)
      CALL CADD(FR(K),FI(K),TMPR,TMPI)
      CALL CM(XR(K+1),XI(K+1),GR(M,1),-GI(M,1)
     + ,TMPR,TMPI)
420   CALL CADD(BR(K),BI(K),TMPR,TMPI)
410   MM=MM+FR(K)*FR(K)+FI(K)*FI(K)+BR(K)*BR(K)+
     + BI(K)*BI(K)
      PM=PM/(2.D0*DFLOAT(NM))
      DO 5 J=1,M
      DO 510 I=1,M
      FGFR(I,1)=0.D0
      FGFI(I,1)=0.D0
      FGMR(I,1)=0.D0
      FGMI(I,1)=0.D0
      IF(I.EQ.J)FGFR(I,1)=1.D0
510   IF(I.EQ.J)FGMR(I,1)=1.D0
      DO 550 I=2,M
      II=I-1
      DO 550 K=1,II
      CALL CM(GR(I-1,I-K),-GI(I-1,I-K),FGMR(I,1),
     + FGMI(I,1),TMPR,TMPI)
      CALL CM(GR(I-1,I),GI(I,I),FGFR(I-1,I-K),-FGMI(I-1,I-K),
     + TMPR,TMPI)
      FGFR(I,K)=FGFR(I-1,K)+TMPR+[illegible]
      FGFI(I,K)=FGFI(I-1,K)+TMPI+TMPI
      CALL CM(GR(I-1,I-K),[illegible]
     + FGMR(I,1),FGMI(I,1),TMPR,TMPI)
      CALL CM(GR(I,I),GI(I,I)
     + FGMR(I-1,I-K),FGMI(I-1,I-K),TMPR,TMPI)
      FGMR(I,K)=FGMR(I-1,K)+TMPR+TMPR
      FGMI(I,K)=FGMI(I-1,K)+TMPI+TMPI
550   CONTINUE
      TMPR=U*DCOS(THETA(J))+U*DCOS(THETA(J+M))
      TMPI=U*DCOS(THETA(J))*DSIN(THETA(J+M))
      TMPR=U*DSIN(THETA(J))*(-DSIN(THETA(J+M)))
      TMPI=U*DSIN(THETA(J))*DCOS(THETA(J+M))
      DO 25 I=1,M
      FGFR(I,1)=TMPR*FGFR(M,I)-TMPI*(-FGMI(M,I))
      FGFI(I,1)=TMPR*FGFI(M,I)+TMPI*FGMR(M,I)
      FGMR(I,1)=TMPR*FGMR(M,I)+TMPI*(-FGMI(M,I))
25    FGMI(I,1)=TMPR*FGMI(M,I)+TMPI*FGMR(M,I)
      FS(J)=0.D0
      FS(J+M)=0.D0
      DO 20 K=1,NM
      T1R=0.D0
      T1I=0.D0
      T2R=0.D0
      T2I=0.D0
      F1R=0.D0
      F1I=0.D0
      F2R=0.D0
      F2I=0.D0
      DO 30 L=1,M
```

```
      CALL CM(XR(K+M-I),-XI(K+M-I),
     + PGPR(L,I),-PGPI(L,I),TMPR,TMPI)
      CALL CADE(T1R,T1I,TMPR,TMPI)
      CALL CM(XR(K+I),-XI(K+I),
     + PGPR(L,I),PGPI(L,I),TMPR,TMPI)
      CALL CADE(T2R,T2I,TMPR,TMPI)
      CALL CM(XR(K+M-I),-XI(K+M-I),
     + PGMR(L,I),-PGMI(L,I),TMPR,TMPI)
      CALL CADE(P1R,P1I,TMPR,TMPI)
      CALL CM(XR(K+I),-XI(K+I),PGMR(L,I),PGMI(L,I),TMPR,TMPI)
30    CALL CADE(P2R,P2I,TMPR,TMPI)
      CALL CM(FR(K),FI(K),T1R,T1I,TMPR,TMPI)
      CALL CM(GR(K),GI(K),T2R,T2I,TMQR,TMQI)
      PS(J)=PS(J)+TMPR+TMQR
      CALL CM(FR(K),FI(K),P1R,P1I,TMPR,TMPI)
      CALL CM(GR(K),GI(K),P2R,P2I,TMQR,TMQI)
20    PS(J+M)=PS(J+M)+TMPR+TMQR
      PS(J)=PS(J)/DFLOAT(MM)
5     PS(J+M)=PS(J+M)/DFLOAT(MM)
      RETURN
      END


      SUBROUTINE CM(XR,XI,YR,YI,ZR,ZI)
      DOUBLE PRECISION XR,XI,YR,YI,ZR,ZI
      ZR=XR*YR-XI*YI
      ZI=XR*YI+XI*YR
      RETURN
      END


      SUBROUTINE CAD(XR,XI,YR,YI,ZR,ZI)
      DOUBLE PRECISION XR,XI,YR,YI,ZR,ZI
      ZR=XR+YR
      ZI=XI+YI
      RETURN
      END


      SUBROUTINE CADE(XR,XI,YR,YI)
      DOUBLE PRECISION XR,XI,YR,YI
      XR=XR+YR
      XI=XI+YI
      RETURN
      END
```

```
      SUBROUTINE [illegible]
      DOUBLE PRECISION H(55),X(20),GRATH(20)
     + ,F,OLDF,GRASUM,T,SOF,DY,HNRM,GNRM,EST,EPS
     + ,FY,ALFA,AMBDA,FX,DX,Z,DALFA,W,DFR,FX1(20),XIS
      COMMON /ZFLEX/L,Z,N,DFR,F,FX [illegible]
      EQUIVALENCE (FX1(20),X(1)),[illegible]
      LUN=5
      IF(LL.EQ.-1)LUN=6
      EPS=10.D0**(-17)
      WRITE(6,2002)
2002  FORMAT([illegible])
      WRITE(LUN,5)
5     FORMAT('   COUNT    RMS GRADIENT    RMS ERROR ')
      CALL FGR(0)
      IER=0
      KOUNT=0
      N2=N+N
      N3=N2+N
      N31=N3+1
10    K=N31
      DO 40 J=1,N
      H(K)=1.D0
      NJ=N-J
      IF(NJ)50,50,20
20    DO 30 L=1,NJ
      KL=K+L
30    H(KL)=0.D0
40    K=KL+1
50    KOUNT=KOUNT+1
      OLDF=F
      GRASUM=0.D0
      DO 90 J=1,N
      GRASUM=GRASUM+GRATH(J)**2
      K=N+J
      H(K)=GRATH(J)
      K=K+N
      H(K)=X(J)
      K=J+N3
      T=0.D0
      DO 80 L=1,N
      T=T-GRATH(L)*H(K)
      IF(L-J)60,70,70
60    K=K+N-L
      GO TO 80
70    K=K+1
80    CONTINUE
90    H(J)=T
      GRASUM=DSQRT(GRASUM/DFLOAT(N))
      SOF=DSQRT(F)
      IF(MOD(KOUNT,5).EQ.0)WRITE(LUN,620)KOUNT,GRASUM,SOF
      DY=0.D0
      HNRM=0.D0
      GNRM=0.D0
      DO 100 J=1,N
      HNRM=HNRM+DABS(H(J))
```

```
      GNRM=GNRM+DABS(GRAD(J))
100   DY=DY+H(J)*GRAD(J)
      IF(DY)110,510,140
110   IF(HNRM/GNRM-EPS)540,540,120
120   FY=F
      ALFA=2.D0*(EST-F)/DY
      AMBDA=1.D0
      IF(ALFA)150,150,130
130   IF(ALFA-AMBDA)140,150,150
140   AMBDA=ALFA
150   ALFA=0.D0
160   FX=FY
      DX=DY
      DO 170 I=1,N
170   X(I)=X(I)+AMBDA*H(I)
      CALL FGR(0)
      FY=F
      DY=0.D0
      DO 180 I=1,N
180   DY=DY+GRAD(I)*H(I)
      IF(DY)190,220,220
190   IF(FY-FX)200,220,220
200   AMBDA=AMBDA+ALFA
      ALFA=AMBDA
      IF(HNRM*AMBDA-1.D10)160,160,210
210   IER=2
      RETURN
220   T=0.D0
230   IF(AMBDA)240,340,240
240   Z=3.D0*(FX-FY)/AMBDA+DX+DY
      ALFA=DMAX1(DABS(Z),DABS(DX),DABS(DY))
      DALFA=Z/ALFA
      DALFA=DALFA*DALFA-DX/ALFA*DY/ALFA
      IF(DALFA)510,250,250
250   W=ALFA*DSQRT(DALFA)
      ALFA=DY-DX+W+W
      IF(ALFA)260,270,260
260   ALFA=(DY-Z+W)/ALFA
      GO TO 280
270   ALFA=(Z+DY-W)/(Z+DX+Z+DY)
280   ALFA=ALFA*AMBDA
      DO 290 I=1,N
290   X(I)=X(I)+(T-ALFA)*H(I)
      CALL FGR(0)
      IF(F-FX)300,300,310
300   IF(F-FY)350,350,310
310   DALFA=0.D0
      DO 320 I=1,N
320   DALFA=DALFA+GRAD(I)*H(I)
      IF(DALFA)330,370,3[illegible]
330   IF(F-FX)3[illegible],3[illegible]0,3[illegible]
340   IF([illegible])
350   FX=[illegible]
      DX=[illegible]
      [illegible]
      [illegible]
      [illegible]
```

```
360     IF(FY-F)380,370,380
370     IF(DY-DALFA)380,390,390
380     FY=F
        DY=DALFA
        AMBDA=AMBDA-ALFA
        GO TO 270
390     IF((OLDF-F)/(OLDF-FPS)-1.0E-20)400,400,400
400     DO 410 J=1,N
        K=N+J
        H(K)=GRAD(J)-H(K)
        K=N+K
410     H(K)=X(J)-H(K)
        IER=0
        IF(KOUNT-N)420,420,420
420     T=0.0
        Z=0.0
        DO 430 J=1,N
        K=N+J
        W=H(K)
        K=K+N
        T=T+ABS(H(K))
430     Z=Z+W*H(K)
        IF(HNRM-EPS)440,440,450
440     IF(T-EPS)550,550,450
450     IF(KOUNT-LIMIT)460,530,530
460     ALFA=0.0
        DO 500 J=1,N
        K=J+N3
        W=0.0
        DO 490 L=1,N
        KL=N+L
        W=W+H(KL)*H(K)
        IF(L-J)470,480,480
470     K=K+N-L
        GO TO 490
480     K=K+1
490     CONTINUE
        K=N+J
        ALFA=ALFA+W*H(K)
500     H(J)=W
        IF(Z*ALFA)510,10,510
510     K=N31
        DO 520 L=1,N
        KL=N2+L
        DO 520 J=L,N
        NJ=N2+J
        H(K)=H(K)+H(KL)*H(NJ)/Z-H(L)*H(J)/ALFA
520     K=K+1
        GO TO 50
530     IER=1
        RETURN
550     DO 560 J=1,N
        K=N2+J
560     X(J)=H(K)
        CALL FUNC(X)
        IF(GNRM-EPS)580,580,570
570     IF(IER.LT.-1) GO TO 580
        IER=IER-1
        GO TO 10
```

```
 580      IER=0
 585      FORMAT(' DIVERGENCE OCCURRED IN ERR [illegible]')
          WRITE(LUN,580)
 5[illegible]      FORMAT(1A,1E20.[illegible])
 590      RETURN
          END

          DOUBLE PRECISION FUNCTION DFLOAT(I)
          DFLOAT=DBLE(FLOAT(I))
          RETURN
          END

          SUBROUTINE MESH
          DOUBLE PRECISION GR(10,10),GI([illegible]),AR(10),[illegible](10)
      *   ,EXTA(320),U,PH,POWER
          COMMON /SLEO/LY,IA,U,PH,POWER/REX/EXTA
          EQUIVALENCE (EXTA(1),GR(1,1)),(EXTA(10[illegible]),GI([illegible]))
      *   ,(EXTA(201),AR(1)),(EXTA(211),AI(1))
          WRITE(5,289)
 289      FORMAT(3X,///,' LY,IA,U,PH,POWER',//)
          WRITE(5,251)LY,IA,U,PH,POWER
 251      FORMAT(2X,[illegible],[illegible])
          WRITE(5,991)
 991      FORMAT(3X,///,' THE DIAGONAL ELEMENTS OF [illegible]')
          WRITE(5,299)(GR(LJ,LJ),GI(LJ,LJ),LJ=1,IA)
 299      FORMAT([illegible](1X,4(D1[illegible],5X,D1[illegible],5,2H ,[illegible])
          WRITE(5,378)
 378      FORMAT(3X,//,' THE [illegible]')
          WRITE(5,299)(AR(LJ),AI(LJ),LJ=1,IA)
          RETURN
          END
```

Appendix IB Burg's Complex Signal MESA Computer Program

```
FORTRAN V06.13                    00:30:14    25-MAR-81    PAGE    1

      C       NAME: CMPBUG [11,11]       DATE: 14-AUG-80
              COMMON INDEX
              REAL F(32),PEF(32),PER(32),FNEW(32),GNEW(32)
              REAL G(20),GGG(20),GGGM(20),HALF(20),H(20)
              REAL SF(513),S(513),YX(1000),CAL(1000)
              COMPLEX F,PEF,PER,GGG,GGGM,G,HALF,H
              WRITE(6,5)
      5       FORMAT(1X,'***** I/P NUM,LG,SN,EXPAND,DT *****'/'0')
              READ(6,10)NUM,LG,SN,EXPAND,DT
      10      FORMAT(2I5,3F12.5)
              WRITE(6,15)
      15      FORMAT(1X,'***** I/P ISTART,ISTOP,INC,NOUT,IZ,LP *****'/'0')
              READ(6,20)ISTART,ISTOP,INC,NOUT,IZ,LP
      20      FORMAT(6I5)
              WRITE(5,25)NUM,LG,ISTART,ISTOP,INC,NOUT,IZ,SN,EXPAND,DT
      25      FORMAT(1H1,5X,7I7,3F12.5)
              CALL BELL
              NI=NUM*2
              DEFINE FILE 2(2,NI,U,INDEX)
              CALL INPUT(F,NUM,SN,FNEW,GNEW)
              WRITE(5,30)NUM
      30      FORMAT(30X,' F  DATA :     NO. OF PTS = ',I5)
              CALL WR(NUM,F,5)
              TPI=8.*ATAN(1.)
              CALL BELL
      50      CONTINUE
              DEN=FLOAT(NOUT-1)*EXPAND
              KFUL=INT(DEN/EXPAND-1.)
              DO 60 KF=1,KFUL
              SF(KF)=FLOAT(KF-1)/DEN-.5
      60      SF(KF)=TPI*SF(KF)
              DO 65 I=1,NUM
              FNEW(I)=REAL(F(I))
              GNEW(I)=AIMAG(F(I))
      65      CONTINUE
              DO 70 I=1,NUM
      70      YX(I)=FLOAT(I)
              CALL NEWPAG
              CALL KBPLOT(YX,FNEW,NUM,50,1010,50,700)
              CALL FIG1(YX,CAL,348,3)
              CALL BELL
              WRITE(6,998)
      998     FORMAT(20X,'***** I/P CHANNEL I    WAVE  *****')
              READ(6,999)NX
      999     FORMAT(I5)
              CALL NEWPAG
              CALL KBPLOT(YX,GNEW,NUM,50,1010,50,700)
              CALL FIG1(YX,CAL,348,3)
              CALL BELL
              WRITE(6,997)
      997     FORMAT(20X,'***** I/P CHANNEL II   WAVE *****')
              READ(6,999)NX
              P=0.
              DO 80 I=1,NUM
      80      P=P+F(I)*CONJG(F(I))
```

```
                 P=P/FLOAT(NUM)
                 SAVE=P
                 WRITE(5,85)P
        85       FORMAT(21X,'P =',E15.7)
                 LA1=NUM-1
                 DO 150 NN=1,LA1
                 NNN=NN+1
                 CALL BPEC(NUM,NN,PEF,PER,F,G,GGG,GGGM,H)
                 P=(1.-G(NN)*CONJG(G(NN)))*P
                 HELP=P
        100      IF(NNN.GT.ISTOP)GO TO 150
                 IF(NNN.LT.ISTART)GO TO 150
                 IF(MOD((NNN-ISTART),INC).NE.0)GO TO 150
                 CALL SPEC(NN,G,S,SF,KFUL,P,DT)
                 CALL BELL
                 CALL BELL
                 READ(6,999)NX
                 CALL NEWPAG
                 CALL KBPLOT(YX,S,NOUT,50,1010,50,700)
                 WRITE(6,120)NNN
                 WRITE(5,120)NN
        120      FORMAT(30X,'***** NO. OF WEIGHT = ',I3,'*****')
                 CALL WR(NN,G,5)
                 CALL FIG1(YX,CAL,I2,2)
                 IF(IP.EQ.1)CALL PRIT(NOUT,DT,S,YX)
                 IF(LG.LE.0)GO TO 140
                 READ(6,999)NX
                 CALL ALG(S,NOUT)
                 CALL NEWPAG
                 CALL WR(NN,G,5)
                 CALL KBPLOT(YX,S,NOUT,50,1010,50,700)
                 WRITE(6,120)NNN
                 CALL FIG1(YX,CAL,I2,1)
        140      P=HELP
        150      CONTINUE
                 CALL BELL
                 CALL EXIT
                 END

        ROUTINES CALLED:
        BELL  , INPUT , WR     , ATAN   , FLOAT , INT    , REAL
        AIMAG , NEWPAG, KBPLOT, FIG1   , CONJG , BPEC   , MOD
        SPEC  , PRIT  , ALG    , EXIT

        OPTIONS =/OP:2

        BLOCK        LENGTH
        MAIN    8185    (037762)*
        .$$$$.  2       (000004)

        **COMPILER ----- CORE**
           PHASE       USED  FREE
        DECLARATIVES 00866 14512
        EXECUTABLES  01263 14115
        ASSEMBLY     01774 18244
```

```
FORTRAN V06.13                     00:31:44     25-MAR-81    PAGE     1

              SUBROUTINE INPUT(X,N1,SN,Y,Z)
              COMMON INDEX
              COMPLEX X(1)
              REAL Y(N1),Z(N1)
              INDEX=1
              READ(2'INDEX)Y
              INDEX=2
              READ(2'INDEX)Z
              DO 5 J=1,N1
              Z(J)=-Z(J)
       5      CONTINUE
              DO 10 I=1,N1
              X(I)=CMPLX(Y(I),Z(I))+SN*CMPLX(GAUSS(1.,0.),GAUSS(1.,0.))
       10     CONTINUE
              WRITE(5,20)
       20     FORMAT(//28X,'CHANNEL I',5X,'CHANNEL II')
              DO 40 I=1,N1
              WRITE(5,30)Y(I),Z(I)
       30     FORMAT(25X,F9.3,5X,F9.3)
       40     CONTINUE
              RETURN
              END

       ROUTINES CALLED:
       CMPLX  , GAUSS

       OPTIONS =/OP:2

       BLOCK         LENGTH
       INPUT    273      (001042)*
       .$$$$.   2        (000004)

       **COMPILER ----- CORE**
          PHASE        USED  FREE
       DECLARATIVES 00627 14756
       EXECUTABLES  00784 14592
       ASSEMBLY     01218 18800
```

```
ORTRAN V06.13                        00:32:11     25-MAR-81    PAGE    1

          SUBROUTINE WR(NN,G,IC)
          REAL G(1)
          COMPLEX G
          DO 10 I=1,NN
          A=REAL(G(I))
          B=AIMAG(G(I))
          IF(IC.EQ.5)WRITE(5,20)A,B
          IF(IC.EQ.6)WRITE(6,20)A,B
    20    FORMAT(28X,'(',F15.7,',',F15.7,')')
    10    CONTINUE
          RETURN
          END

    ROUTINES CALLED:
    REAL   , AIMAG

    OPTIONS =/OP:2

    BLOCK         LENGTH
    WR       150     (000454)*

    **COMPILER ----- CORE**
       PHASE         USED  FREE
    DECLARATIVES  00702 14676
    EXECUTABLES   00783 14595
    ASSEMBLY      01097 18921


FORTRAN V06.13                        00:32:31     25-MAR-81    PAGE    1

          FUNCTION GAUSS(XBAR,SIGMA)
          ROOT=1./SQRT(8.*ATAN(1.))
    10    CONTINUE
          X=10.*(RAN(0,1)-.5)
          Y=ROOT*EXP(-X*X/2.)
          YTRY=ROOT*RAN(0,1)
          IF(YTRY.GT.Y)GO TO 10
          GAUSS=X*SIGMA+XBAR
          RETURN
          END

    ROUTINES CALLED:
    SQRT   , ATAN   , RAN    , EXP

    OPTIONS =/OP:2

    BLOCK         LENGTH
    GAUSS    171     (000526)*

    **COMPILER ----- CORE**
       PHASE         USED  FREE
    DECLARATIVES  00672 14706
    EXECUTABLES   00783 14595
    ASSEMBLY      01033 18985
```

```
FORTRAN V06.13                    00:32:54     25-MAR-81     PAGE      1

              SUBROUTINE BPEC(NUM,NN,PEF,PER,F,G,GGG,GGGM,H)
              DIMENSION H(1),PEF(1),PER(1),F(1),G(1),GGG(1),GGGM(1)
              COMPLEX G,PEF,PER,H,GGG,GGGM,F,P,Q,SN,SD
              XERO=(0.,0.)
              N=NN-1
              IF(N.NE.0)GO TO 20
              DO 10 J=1,NUM
              PEF(J)=XERO
     10       PER(J)=XERO
     20       SN=XERO
              SD=XERO
              JJ=NUM-N-1
              DO 30 J=1,JJ
              Q=F(J+N-1)-PEF(J)
              P=F(J)+PER(J)
              SN=SN+CONJG(P)*Q
              SD=SD+P*CONJG(P)+Q*CONJG(Q)
     30       CONTINUE
              GGG(NN)=-2.*SN/SD
              IF(N.EQ.0)GGGM(1)=GGG(1)
              IF(N.EQ.0)GO TO 60
              DO 40 J=1,N
              K=N-J-1
              H(J)=GGG(J)+GGG(NN)*CONJG(GGG(K))
     40       CONTINUE
              DO 50 J=1,N
     50       GGG(J)=H(J)
              GGGM(NN)=GGG(NN)
              JJ=JJ-1
     60       DO 70 J=1,JJ
              PER(J)=PER(J)+CONJG(GGG(NN))*(PEF(J)+F(J+NN))
              PEF(J)=PEF(J+1)+GGG(NN)*(PER(J+1)+F(J+1))
     70       CONTINUE
              DO 80 I=1,NN
     80       G(I)=GGG(I)
              RETURN
              END

     ROUTINES CALLED:
     CONJG

     OPTIONS =/OP:2

     BLOCK        LENGTH
     BPEC     523     (002026)*

     **COMPILER ----- CORE**
        PHASE        USED   FREE
     DECLARATIVES 00622 14756
     EXECUTABLES  00863 14515
     ASSEMBLY     01205 18813
```

```
      SUBROUTINE SPEC(NN,G,S,SF,KFUL,P,DT)
      DIMENSION S(1),SF(1),G(1)
      COMPLEX G,SCS,ONE,GK
      ONE=(1.,0.)
      WRITE(5,5)P
5     FORMAT(15X,'***** M.E.P= ',E15.7,'  *****')
      PQ=2.*P*DT
      DO 20 KF=1,KFUL
      CA=1.
      SA=0.
      SCS=ONE
      ADD=SF(KF)
      CT=COS(ADD)
      ST=SIN(ADD)
      DO 10 K=1,NN
      GK=G(K)
      TEMP=CT*CA-ST*SA
      SA=CT*SA+ST*CA
      CA=TEMP
      SCS=SCS+GK*CMPLX(CA,SA)
10    CONTINUE
      S(KF)=PQ/(SCS*CONJG(SCS))
20    CONTINUE
      RETURN
      END
```

```
ROUTINES CALLED:
COS    , SIN    , CMPLX , CONJG

OPTIONS =/OP:2

BLOCK        LENGTH
SPEC    305    (00164)*

**COMPILER ----- CORE**
   PHASE        USED  FREE
DECLARATIVES  00622 14756
EXECUTABLES   00943 14435
ASSEMBLY      01273 18745
```

```
FORTRAN V06.13                    00:34:02    25-MAR-81    PAGE    1

                SUBROUTINE PRIT(NOUT,DT,S,YX)
                REAL S(1),YX(1)
                FRE01=1./FLOAT(NOUT-1)/DT
                YX(1)=-0.50
                DO 10 I=2,NOUT
        10      YX(I)=YX(I-1)+FRE01
                DO 30 J=1,NOUT
                WRITE(5,20)J,YX(J),S(J)
        20      FORMAT(10X,I3,5X,E15.7,5X,E15.7)
        30      CONTINUE
                RETURN
                END

        ROUTINES CALLED:
        FLOAT

        OPTIONS =/OP:2

        BLOCK        LENGTH
        PRIT     173     (000532)*

        **COMPILER ----- CORE**
           PHASE        USED  FREE
        DECLARATIVES 00672 14756
        EXECUTABLES  00783 14595
        ASSEMBLY     01141 18877
```

```
ORTRAN V06.13                          00:34:23     25-MAR-81    PAGE     :

                 SUBROUTINE FIG1(YX,CAL,I2,KG)
                 REAL CAL(1),YX(1)
                 CAL(1)=330.
                 DO 100 I=1,960
                 IF(I.EQ.1)GO TO 100
                 I1=I-I/I2*I2
                 IF(I1.EQ.0)GO TO 50
                 CAL(I)=0.
                 GO TO 100
       50        CAL(I)=5.
       100       CONTINUE
                 DO 200 J=1,960
       200       YX(J)=FLOAT(J)
                 CALL KBPLOT(YX,CAL,960,50,1010,50,725)
                 IF(KG.EQ.2)WRITE(6,205)
       205       FORMAT(5X,'LINEAR SCALE')
                 IF(KG.EQ.1)WRITE(6,210)
       210       FORMAT(5X,'LOG  SCALE')
                 IF(KG.EQ.3)WRITE(6,215)
       215       FORMAT(20X,'TIME  SCALE')
                 CALL BELL
                 RETURN
                 END

       ROUTINES CALLED:
       FLOAT , KBPLOT, BELL

       OPTIONS =/OP:2

       BLOCK          LENGTH
       FIG1     285     (001072)*

       **COMPILER ----- CORE**
          PHASE        USED  FREE
       DECLARATIVES 00622 14756
       EXECUTABLES  00863 14515
       ASSEMBLY     01177 18541
```

```
ORTRAN V06.13                        00:34:53     25-MAR-81    PAGE     1

                    SUBROUTINE ALG(S,KFUL)
                    REAL S(1)
                    DO 10 I=1,KFUL
                    IF(S(I).GT.0.)S(I)=ALOG10(S(I))
          10        CONTINUE
                    RETURN
                    END

          ROUTINES CALLED:
          ALOG10

          OPTIONS =/OP:2

          BLOCK         LENGTH
          ALG       74       (000224)*

          **COMPILER ----- CORE**
             PHASE         USED  FREE
          DECLARATIVES 00703 14676
          EXECUTABLES  00702 14676
          ASSEMBLY     00943 19077
```

Appendix II

On Multichannel (Multivariate) Maximum Entropy Spectral Analysis

1. Introduction

The univariate maximum entropy spectral analysis has now been well developed and applied to many defense research areas such as radar, sonar and geophysics. There has been some work done to extend the maximum entropy spectral analysis to multivariate case. Whittle [1] and Robinson [2][3] generalized the Levinson-Durbin recursion to the multivariate case by fitting both foward and backward autoregressions in a stepwise fashion. In this thesis, Burg [4] has mentioned about the multichannel case. However the computer programs for both multichannel and multivariate maximum entropy spectral analysis were only recently developed successfully. Morf et.al [5] developed an algorithm for direct estimation of the normalized reflection coefficients from the observed data for maximum entropy spectral analysis. They also compared the spectral estimation with the methods of Jones [6], Nuttall [7] and Strand [8], which are more of a direct extension of Burg's work to the multichannel (multivariate) case. Burg's algorithm does not generalize directly since the forward and backward autoregression matrices are not the same in the multivariate case, and the forward and backward one-step prediction error covariance matrices are different, although they have the same determinant. In this report, the programs developed by Strand and Jones are applied to real multichannel data and imagery data in addition to a set of test signals. The merits of these methods are closely examined. In spite of programming complexity the multichannel and multivariate maximum entropy spectral analysis will have increased application as the real data are almost always gathered in several channels. Data from several channels form a vector for multivariate study.

2. Brief Mathematical Analysis

Let $x_1, x_2, \ldots, x_n$ denote n zero mean vectors of dimension d each. The sample estimate of covariance sequence for lag j is

$$\hat{R}_j = \frac{1}{n} \sum_{t=1}^{n-j} x_{t+j} x_t' \tag{1}$$

where the prime denotes the transposed vector. The forward and backward predicting autoregressions of order p are given, respectively, as

$$\begin{aligned} \hat{x}_t^{(f)} &= \sum_{k=1}^{p} A_k^{(p)} x_{t-k} \\ \hat{x}_t^{(b)} &= \sum_{k=1}^{p} B_k^{(p)} x_{t+k} \end{aligned} \tag{2}$$

where $A_k^{(p)}$ and $B_k^{(p)}$ are d x d matrices, and can be determined recursively [6] by making use of the estimated covariance matrix in Eq. (1). The recursion starts with

$$S_o^{(f)} = S_o^{(b)} = R_o \tag{3}$$

The one-step forward and backward prediction error covariance matrices are

$$\begin{aligned} S_p^{(f)} &= (I - A_p^{(p)} B_p^{(p)}) S_{p-1}^{(f)} \\ S_p^{(b)} &= (I - B_p^{(p)} A_p^{(p)}) S_{p-1}^{(b)} \end{aligned} \tag{4}$$

The forward and backward residuals are, respectively,

$$\begin{aligned} e_t^{(p)} &= x_t - \sum_{k=1}^{p} A_k^{(p)} x_{t-k}, \quad t = p+1,\dots,n \\ \beta_t^{(p)} &= x_t - \sum_{k=1}^{p} B_k^{(p)} x_{t+k}, \quad t = 1,\dots,n-p \end{aligned} \tag{5}$$

The recursive equations are then given by

$$\begin{aligned} e_t^{(p)} &= e_t^{(p-1)} - A_p^{(p)} \beta_{t-p}^{(p-1)}, \quad t = p+1,\dots,n \\ \beta_t^{(p)} &= \beta_t^{(p-1)} - B_p^{(p)} e_{t+p}^{(p-1)}, \quad t = 1,\dots,n-p \end{aligned} \tag{6}$$

The least squares estimates for the forward and backward autoregression matrices are

$$\begin{aligned} A_p^{(p)} &= UV^{-1} \\ B_p^{(p)} &= U'W^{-1} \end{aligned} \tag{7}$$

where U is the sum of cross products of forward and backward residuals at lag p,

$$U = \sum_{t=1}^{n-p} e_{t+p}^{(p-1)} (\beta_t^{(p-1)})' \tag{8}$$

and V and W are estimates of $(n - p)S_{p-1}^{(b)}$, $(n - p)S_{p-1}^{(f)}$ respectively,

$$V = \sum_{t=1}^{n-p} \beta_t^{(p-1)} (\beta_t^{(p-1)})' \tag{9}$$

$$W = \sum_{t=1}^{n-p} e_{t+p}^{(p-1)} (e_{t+p}^{(p-1)})' \tag{10}$$

Although the forward and backward autoregression matrices and the prediction error covariance matrices are different, the multivariate spectra should be identical when calculated from the forward and backward fits by

$$S(f) = h[A(f)]^{-1} S_p^{(f)} [A^*(f)]^{-1}$$

or by

$$S(f) = h[B(f)]^{-1} S_p^{(b)} [B^*(f)]^{-1}$$

where

$$A(f) = I - \sum_{k=1}^{p} A_k^{(p)} e^{2\pi ikhf}$$

$$B(f) = I - \sum_{k=1}^{p} B_k^{(p)} e^{-2\pi ikhf}$$

h is the sampling period and * denotes complex conjugate transpose.

The above approach based on the work of Jones [6] does not guarantee stability and does not generally produce a non-negative definite spectrum as has been pointed out by Nuttall [7]. Subsequently, Nuttall [7] and Strand [8] applied a weighted arithmetic mean error criterion in order to provide model stability and to ensure positive definite stationary spectra. Another procedure suggested by Morf, et.al. [5] that also meets the spectral requirements is to compute the spectrum from the normalized reflection coefficient matrix ρ. To obtain this matrix, W and V are factored by using Cholesky decomposition into the product of lower triangular matrices times their transposes. A new recursive procedure for $S_p^{(f)}$ and $S_p^{(b)}$ by using ρ in place of Eqs. (4) can then be obtained. Other recursive algorithm has been proposed [9] for the solution of the normal equations for both single and multichannel data.

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Appendix III

A BIBLIOGRAPHY ON MAXIMUM ENTROPY SPECTRAL ANALYSIS AND RELATED TECHNIQUES

I. Introduction

Recently there has been strong research interest on high resolution spectral analysis techniques. This is an important area of defense research because of the numerous applications to radar, sonar, and geophysical areas of defense interest. An excellent publication is the Proceedings of the 1978 and 1979 RADC Spectrum Estimation Workshop. Maximum entropy spectral analysis is one of a number of high resolution spectral analysis techniques. The impact of the Burg's maximum entropy spectral analysis method is far more significant than the technique itself. Thus in this report we present not only the bibliography of the maximum entropy methods in one and two spatial dimensions but also a number of related methods of high resolution spectral analysis. One common assumption with all these methods is that the data record is short and thus the conventional fast Fouriter transform method of spectral analysis is not suitable. Probably because of the short length record, the maximum entropy spectral computation is fairly sensitive to the presence of noise. In the following sections, references are arranged in the first author's alphabetical order. Each reference is listed only once in the report. Effort has been made to provide as complete list of references as possible.

II. Bibliography on Univariate Maximum Entropy Spectral Analysis

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